CBCS SCHEME

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17CS/IS36

Third Semester B.E. Degree Examination, July/August 2022
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. State and prove Distributive Laws of Logic, using truth table. **(05 Marks)**

b. Test the validity of the following argument:

If Ravi goes out with friends, he will not study

If Ravi does not study, his father becomes angry

His father is not angry

$\therefore$ Ravi has not gone out with friends **(05 Marks)**

c. Determine the truth value of the following statements if the universe comprises all nonzero integers.

(i) $\exists x\, \exists y\, [xy = 2]$ (ii) $\exists x\, \forall y\, [xy = 2]$ (iii) $\forall x\, \exists y\, (xy = 2)$

(iv) $\exists x\, \exists y\, ((3x - y = 8) \wedge (2x - y) = 7))$ (v) $\exists x\, \exists y\, ((4x + 2y = 3) \wedge (x - y = 1))$ **(05 Marks)**

d. Give : (i) Direct proof (ii) Proof by contradiction for the following statement.
If n is an odd integer, then $(n + 9)$ is an even integer. **(05 Marks)**

OR

2 a. Prove that $((A \wedge B) \to C) \Leftrightarrow (A \to (B \to C))$ is a tautology. **(05 Marks)**

b. Establish the validity of the following argument by method of contradiction:

$p \to (q \wedge r)$

$r \to s$

$\neg(q \wedge s)$

$\therefore \neg p$ **(05 Marks)**

c. Define converse, inverse, contrapositive of implication $p \to q$. Give example for each. **(05 Marks)**

d. Find whether following argument is valid. Universe is sit of all triangles.

If a traingle has 2 equal sides, it is isoceles

If a triangle is isoceles, it has 2 equal angles

A certain traingle ABC does not have 2 equal angles

$\therefore$ Triangle ABC does not have 2 equal sides **(05 Marks)**

Module-2

3 a. Prove by mathematical induction that

$$(1\times 2) + (2\times 3) + (3\times 4) + \ldots\ldots + (n\times(n+1)) = \frac{1}{3}n(n+1)(n+2) \text{ where } n \ge 1$$ **(05 Marks)**

b. A sequence $\{a_n\}$ is defined $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \ge 2$. Find a_n in explicit form. **(05 Marks)**

c. How many arrangements are there for all letters in the word SOCIOLOGICAL. In how many of the arrangements (i) A and G are adjacent (ii) All vowels are adjacent. **(05 Marks)**

d. In how many ways can we distribute 8 identical balls into 4 distinct containers so that :

(i) no container is left empty

(ii) The 4^{th} container gets an odd number of balls **(05 Marks)**

OR

4 a. (i) Find the number of 3-digit even numbers with no repeated digits.
(ii) In how many ways can we distribute 7 apples and 6 oranges among 4 children so that each child gets atleast one apple. **(05 Marks)**

b. Find the coefficient of
(i) x^9y^3 in the expansion of $(2x-3y)^{12}$
(ii) xyz^2 in the expansion of $(2x-y-z)^4$ **(05 Marks)**

c. A certain question paper contains 3 parts A, B, C, with 4 questions in part A, 5 questions in part B and 6 questions in part C. It is required to answer 7 questions selecting at least 2 questions from each part. In how many different ways can a student select his 7 questions for answering? **(05 Marks)**

d. Find the number of arrangements of the letters in the word TALLAHASSEE. How many of these arrangements have no adjacent A's? **(05 Marks)**

Module-3

5 a. Let $f: R \to R$ be defined by $f(x) = \begin{cases} 3x-5, & \text{for } x > 0 \\ -3x+1, & \text{for } x \le 0 \end{cases}$. Determine $f^{-1}(0)$, $f^{-1}(1)$, $f^{-1}(-1)$, $f^{-1}(-3)$, $f^{-1}(-6)$. **(05 Marks)**

b. Let $A = \{a, b, c, d\}$, $B = \{1, 2, 3, 4, 5, 6\}$,
(i) How many functions are there from A to B? How many of these are one-one and how many are onto?
(ii) How many functions are there from B to A? How many of these are one-to-one and how many are onto? **(05 Marks)**

c. Let $A = \{1, 2, 3, 4, 6, 8, 12\}$. On A, define a relation R by x R y if and only if x divides y. Write ordered pairs of R and show that R is a partial ordering relation. Draw Hasse diagram of R. **(05 Marks)**

d. Define Reflexive, symmetric, transitive, antisymmetric, equivalence relation. **(05 Marks)**

OR

6 a. Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$. Determine:
(i) $|A \times B|$
(ii) Number of relations from A to B
(iii) Number of relations from A to B that contain (1, 2) and (1, 5)
(iv) Number of relations from A to B that contain exactly 5 ordered pairs
(v) Number of binary relations on A that contain at least 7 ordered pairs. **(05 Marks)**

b. Justify using Pigenhole principle:
(i) Any subset of size 6 from the set $A = \{1, 2, 3, \ldots, 9\}$ must contain at least 2 elements whose sum is 10.
(ii) Wilma operates a computer with a magnetic tape drive. One day she is given a tape that contains 500000 words of 4 or fewer lowercase letters. Can it be that all 500000 words are all distinct? **(05 Marks)**

c. Let f, g, h functions from z to z defined by $f(x) = x - 1$, $g(x) = 3x$,

$$h(x) = \begin{cases} 0, & \text{if } x \text{ is even} \\ 1, & \text{if } x \text{ is odd} \end{cases}$$

Determine $(f \circ (g \circ h))(x)$ and $((f \circ g) \circ h)(x)$. **(05 Marks)**

d. On the set z, a relation R is defined by a R b if and only if $a^2 = b^2$. Verify that R is an equivalence relation. Determine the partition induced by R. **(05 Marks)**

Module-4

7 a. Find the number integers between 1 and 10,000 inclusive, which are divisible by none of 5, 6, or 8. **(08 Marks)**

b. What is derangement? Find the number of derangements of 1, 2, 3, 4 and list these derangements. **(06 Marks)**

c. Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$ where $n \geq 0$ and $F_0 = 0$, $F_1 = 1$. **(06 Marks)**

OR

8 a. Find the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 18$ under the condition $x_i \leq 7$, for i = 1, 2, 3, 4. **(08 Marks)**

b. A person invests Rs.1,00,000 at 12% interest compounded annually:

(i) Find the amount at the end of 1st, 2nd, 3rd year.

(ii) Write the general explicit formula

(iii) How long will it take to double the investment? **(06 Marks)**

c. Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 2$, given that $a_1 = 5$ and $a_2 = 3$. **(06 Marks)**

Module-5

9 a. Define the following with an example for each:

(i) Connected graph (ii) Complete graph (iii) Regular graph
(iv) Bipartite graph (v) Complete bipartite graph (vi) Euler graph **(06 Marks)**

b. Determine order |V| of G = (V, E) if

(i) G is a cubic graph with 9 edges

(ii) G is Regular with 15 edges

(iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3. **(04 Marks)**

c. Define isomorphism. Show that following graphs, shown in Fig.Q9(c)(i) and (ii) are isomorphic.

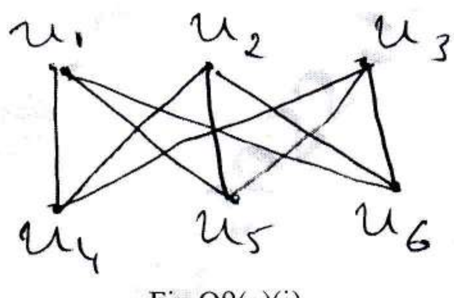

Fig.Q9(c)(i)

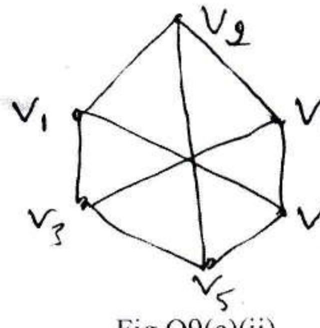

Fig.Q9(c)(ii) **(04 Marks)**

d. Explain about Konigsberg Bridge Problem and about its solution. **(06 Marks)**

OR

10 a. Define walk, trail, path, circuit, cycle, degree of a vertex in a graph, with an example for each. **(06 Marks)**

b. Prove that in every graph, the number of vertices of odd degree is even. **(04 Marks)**

c. Prove that in every tree T = (V, E), |V| = |E| + 1. **(04 Marks)**

d. Construct an optimal tree for a given set of weights, {4, 15, 25, 5, 8, 16}. Hence find weight of the optimal tree. **(06 Marks)**

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